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The University of Southern Mississippi

FINDING THE LIMITATIONS OF THE EXPANDED EQUIVALENT FLUID
APPROXIMATION FOR SIMULATING ACOUSTIC INTERACTIONS WITH THE
OCEAN BOTTOM

by

Steven J. Kirkup

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Abstract

Equivalent fluids use complex densities in place of realistic seafloor conditions to simplify simulations of acoustic interaction with the seafloor. This eliminates the computationally intensive attempts to simulate realistic seafloor interactions that include shear waves. A previous method used such equivalent fluids and was found to be accurate only for interaction with low grazing angles and low shear speeds. The current method expands by also parameterizing the speed of sound in the fluid, allowing higher grazing angles and shear speeds to be modeled with equivalent fluids. For a particular window of grazing angle, there are several approaches to determining the complex density and the fluid's speed of sound using this Expanded Equivalent Fluid. By calculating statistics and comparing acoustic simulation results, the approach that most accurately mimics the actual seafloor and the highest window of grazing angle at which the fluid yields respectable results are investigated.

Key Terms: acoustical oceanography, underwater acoustics, marine geology, numerical methods, acoustic remote sensing, computer simulation

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Chapter 1: Introduction

In the realm of ocean acoustic tomography, various properties of a body of water can be determined by transmitting acoustic signals through the body and interpreting the signal recorded at a receiving end. These properties include temperature, currents, salinity, and pressure. One difficulty experienced with this technique is properly interpreting the received data, as the geography of the ocean bottom (henceforth called “bathymetry”) can complicate that data if the water is shallow enough that the signal interacts with the seafloor.

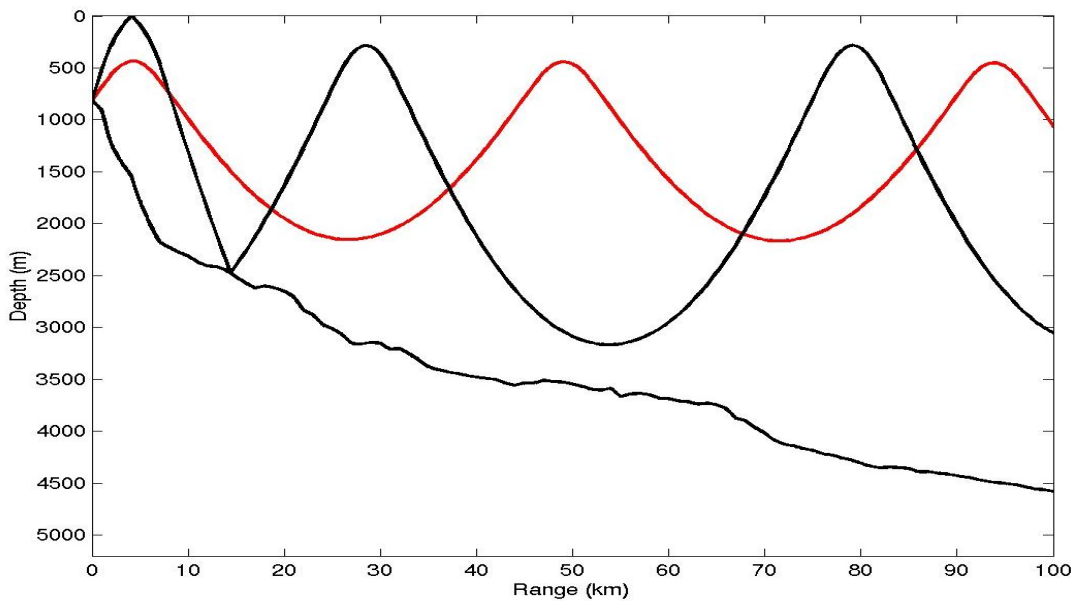


Figure 1—This is an example of sound rays propagating through water. One interacts with the seafloor, complicating the signal.

The bottom solid black line depicts the bathymetry recorded off the coast of Kauai. (Vera *et al.* 2005).

Steps are often taken to minimize this issue—namely, transmitting and recording the signals offshore in deep water. However, in experiments where the acoustic-bathymetry interaction (“bottom interaction” for short) cannot be avoided, an understanding of how the ocean bottom affects the signal is needed.

This understanding can be achieved by engineering a model of the ocean bottom that embodies the bottom’s characteristics that specifically affect incoming acoustic signals. If the

model can be used to accurately replicate experimental results, the possibility that the model can be used for that bottom type and similar types is considered. By finding the range of parameters for which the model still accurately approximates bottom interaction, researchers can test the model's viability for experiments within said parameters.

The Expanded Equivalent Fluid (EEF) approximation is the model investigated in this study. As the “expanded” in the name suggests, EEF is a more versatile edition of a previous fluid approximation, and its limitations are yet to be found. The aim of this research is to determine the precise versatility of this expanded model.

Chapter 2: Literature Review

The formulation of the Expanded Equivalent Fluid approximation has evolved over many years of study. The aforementioned issue of a complicated bottom interaction led researchers Zhang and Tindle to develop their first model of what they called an “equivalent fluid.”

When a sound wave encounters a solid, there are three resulting waves: a reflected sound wave into the original medium, a refracted sound wave into the solid, and a transverse shear wave into the solid. The shear wave only occurs in the solid due to the solid's elastic restoring force, a property fluids do not have (Feynman 1977). Zhang and Tindle (1992) state that if the seafloor is considered a homogeneous solid and the ocean a homogeneous liquid, the expression for the reflection coefficient V_{fs} , which is the characteristic of the bottom needed to determine how it alters the acoustic signal, can be written in terms of the ocean's density ρ_1 and sound speed c_1 , the bottom's density ρ_2 and sound speed c_2 , the speed of shear waves in the bottom c_s (where $c_s < c_1 < c_2$), the vertical wave numbers of the ocean, bottom, and shear waves (γ_1 , η_2 , and γ_s respectively), and the function $P(k)$ such that

$$V_{fs}(k) = \frac{\gamma_1 \left[\frac{\rho_2 P(k)}{\rho_1} \right] - i\eta_2}{\gamma_1 \left[\frac{\rho_2 P(k)}{\rho_1} \right] + i\eta_2}$$

where k is the horizontal wave number in terms of angular frequency ω , grazing angle θ between the incident acoustic wave and the bottom surface, and c_1 and is expressed as

$$k = \frac{\omega}{c_1} \cos \theta$$

(Zhang and Tindle 1992). The vertical wave numbers γ_1 , η_2 , and γ_s are equated by the following:

$$\begin{aligned}\gamma_1 &= \sqrt{\left(\frac{\omega}{c_1}\right)^2 - k^2} \\ \eta_2 &= \sqrt{k^2 - \left(\frac{\omega}{c_2} + i\alpha_2\right)^2} \\ \gamma_s &= \sqrt{\left(\frac{\omega}{c_s} + i\alpha_s\right)^2 - k^2}\end{aligned}$$

where α_2 and α_s are the attenuation coefficients of the bottom for the sound and shear waves respectively. Lastly, the parameter $P(k)$ is expressed as

$$P(k) = \left(1 - \frac{2k^2}{\left(\frac{\omega}{c_s} + i\alpha_s\right)^2} \right)^2 + \frac{i4\eta_2\gamma_s k^2}{\left(\frac{\omega}{c_s} + i\alpha_s\right)^4}$$

However, Zhang and Tindle (1992) found that if we consider the bottom a fluid instead, the expression for the reflection coefficient simplifies to

$$V_{ff}(k) = \frac{\gamma_1 \left[\frac{\rho'_2}{\rho_1} \right] - i\eta'_2}{\gamma_1 \left[\frac{\rho'_2}{\rho_1} \right] + i\eta'_2}$$

since this change sets c_s and α_s equal to zero and $P(k) = 1$. The equivalent fluid approximation they came up with involved using V_{ff} in place of V_{fs} and a complex density that allowed V_{ff} to be a near approximation of the more realistic V_{fs} (Zhang and Tindle 1992). They later improved their method, though, and their resulting expression for the density is

$$\rho'_2 = \rho_2 \left[\left(1 - \frac{2}{\left(\frac{c_1}{c_s} + \frac{i\alpha_s c_1}{\omega} \right)^2} \right)^2 + \frac{i4 \sqrt{\left(1 - \left(\frac{c_1}{c_2} + \frac{i\alpha_2 c_1}{\omega} \right)^2 \right) \left(\left(\frac{c_1}{c_s} + \frac{i\alpha_s c_1}{\omega} \right)^2 - 1 \right)}}{\left(\frac{c_1}{c_s} + \frac{i\alpha_s c_1}{\omega} \right)^4} \right]$$

(Zhang and Tindle 1995).

The North Pacific Acoustic Laboratory experiment was one instance in which Zhang and Tindle's equivalent fluid approximation had the potential to be used. In this experiment, an acoustic signal was transmitted into the Pacific Ocean off the shore of the Hawaiian island Kauai, and receivers in multiple locations recorded the signal, including a plane array of receivers off the coast of California and a receiver at Ocean Weather Station Papa located in the Gulf of Alaska (Worcester and Spindle 2005). Since the signal originated in shallow depths, the possibility of the signal experiencing bottom interaction and still reaching a receiver was large enough to warrant use of the equivalent fluid approximation (Vera *et al.* 2005). Figure 1 is of the bathymetry off the shore of Kauai. After reviewing Zhang and Tindle's work, though, Dr. Michael Vera concluded that their model was only accurate at low grazing angles and low shear speed. However, Vera found the model could be expanded to work for larger angles and shear speeds by considering the imaginary and real parts of the complex density and the equivalent fluid sound speed (c_2) as "free parameters" in an attempt to match the seafloor reflection coefficient (Vera *et al.* 2005). This is allowable in the sense that the equivalent fluid is supposed be a mathematical model that mimics the effect the ocean bottom has on acoustic signals; the

characteristics of the equivalent fluid should not necessarily be constrained to realistic values, so long as they yield accurate approximations. Appropriately, Vera's expanded version of the equivalent fluid model has become known as the Expanded Equivalent Fluid (EEF) approximation.

In the POMA article “Complex-density, equivalent-fluid modeling of acoustic interaction with the seafloor,” Vera compared the EEF approximation to Zhang and Tindle's equivalent fluid approximation by plotting their results for a specific case against the Range-dependent Acoustic Modeling with Shear (RAMS) simulation (also called a “benchmark” simulation) (2008); this model computes the effects of shear more realistically, but is therefore more computationally complicated—too complicated to feasibly simulate multi-frequency long-range transmissions (Collins 1994). However, RAMS can accurately simulate transmissions of one frequency, so Vera compared the two equivalent-fluid approximations with the RAMS simulation in this way. Using one of the propagation paths from the NPAL experiment and a 75Hz frequency, Vera ran all three simulations and plotted the results on a transmission loss (in dB) vs. range (in km) graph. This depicts the amount of energy a signal loses against how far away from the signal's origin the receiver is placed. The RAMS simulation, Zhang and Tindle's equivalent fluid approximation, and the EEF approximation are plotted as solid, dotted, and dashed lines respectively in Figure 2 (Vera 2008). Vera states, “These tests...have shown that the [EEF] method can accurately depict seafloor materials with high shear speeds; in fact, the limiting factor on the performance of the method appears to be the size of the relevant grazing-angle interval. Additional work will involve quantifying these limits on grazing angle and the range of bottom parameters that can be reliably modeled.”(Vera 2008).

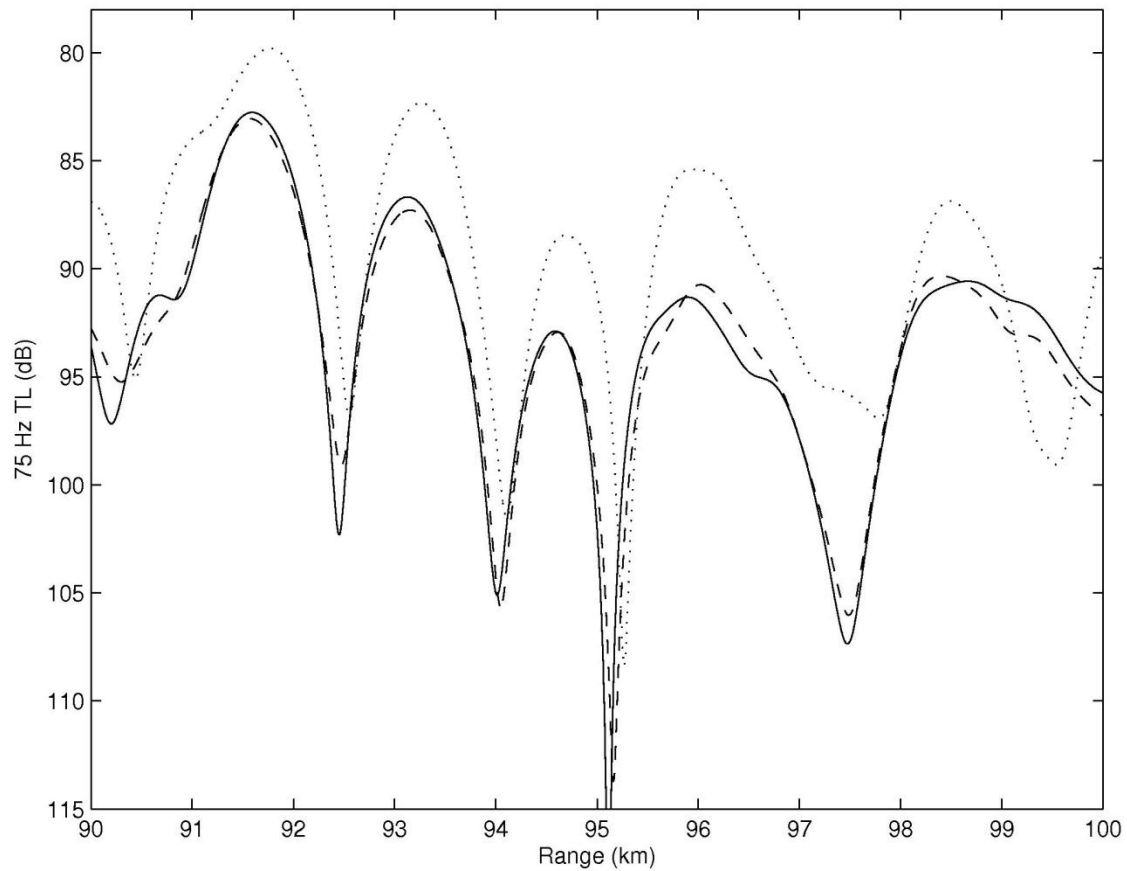


Figure 2—This is a transmission loss vs. range graph of the RAMS, EEF, and the Zhang and Tindle equivalent fluid plotted as solid, dashed, and dotted lines respectively (Vera 2008).

In Kevin Heaney, Michael Vera, and Arthur Baggeroer’s working document “Geoacoustic inversion for a volcanic seafloor using equivalent fluids,” the three describe a method used to determine the values for the real and imaginary densities and seafloor sound speed (the free parameters) of an equivalent fluid that accurately simulates a bottom of unknown properties. They explain that multiple equivalent fluids can be modeled with varying values of the free parameters; the simulated acoustic results using each fluid can be compared to

experimental data to see which is more accurate (Heaney *et al.* 2011). The comparisons were done with what Heaney *et al.* call a “cost function” and is given by

$$C_A = \frac{1}{N_i} \sum_{t_i} |D(t_i) - S(t_i)|$$

where N_i is the number of included values, and $D(t_i)$ and $S(t_i)$ are the dB levels of the experimental data and fluid simulation respectively. For their varying degrees of the free parameters, Heaney *et al.* used 20 values from 1000 to 4800 m/s for the seafloor sound speed, 14 values from 200 to 2800 kg/m³ for the real part of density, and 14 values of the same range for the imaginary part of density, yielding 3920 different equivalent fluid approximations (Heaney *et al.* 2011). Also, the attenuation coefficient was set to 0.5 dB/λ for all approximations, as its deviation is minimal throughout different bottom materials, and changes in it have little impact on the value of the reflection coefficient. They found the case with the smallest cost function value (and therefore the most accurate) to have seafloor sound speed 4200 m/s and density 400 + 200i kg/m³; to determine the properties of the elastic solid, they compare the acoustic bottom loss of this equivalent fluid to the bottom loss of elastic solids. The cost function for this comparison is

$$C_V = \frac{1}{N_i} \sum_{\theta_i} \frac{|L_{es}(\theta_i) - L_{ef}(\theta_i)|}{L_{es}(\theta_i)}$$

where L_{es} and L_{ef} are the bottom losses for the solid bottom and equivalent fluid respectively, calculated by $L = -10 \log |V|^2$, θ_i is a set of integer angles (in this case, from 25 to 65°), and N_i is the total amount of those integer angles (Heaney *et al.* 2011).

In general, the EEF approximation shows importance in advancing techniques in tomography. For the NPAL experiment, the approximation was vital in identifying the acoustic arrivals at the California receivers. For the volcanic seafloor research, the equivalent fluid

methods were found to be helpful in determining the properties of a solid bottom from acoustic data. Another example involves simulations conducted in an environment resembling the Gulf of Mexico: here, sound was transmitted in shallow water and received multiple directions away from the source. The intention was to see if any large amounts of chemicals--oil, for example--could be detected suspended in the sea water (Vera 2010). Again, the EEF approximation aided the interpretation of acoustic signals since bottom interaction is prominent in shallow water (Vera 2010).

Chapter 3: Methods

To find the limitations of the EEF approximation, methods similar to those previously stated were used. Instead of determining the specifics of an equivalent fluid that best mimics a particular bottom of unknown properties, however, this study aims to determine the limits of the EEF method in general. This was done by comparing an existing elastic solid (i.e. a benchmark seafloor) to some equivalent fluids to find the limit of the fluid's accuracy. The method executed took said elastic bottom and compared it to the following: Zhang and Tindle's equivalent fluid; the original expanded equivalent fluid; and two new "versions" of the equivalent fluid (named "Met 1" and "Met 2") that were found using altered metrics. A "metric" is a cost function that the EEF calculation algorithm attempts to minimize. The original metric (C_{orig}) and the new metrics Met 1 (C_{M1}) and Met 2 (C_{M2}) are as follows:

$$C_{orig} = \frac{1}{N_i} \sum_{\theta_i} \frac{|V_{es}(\theta_i) - V_{ef}(\theta_i)|}{|V_{es}(\theta_i)|}$$

$$C_{M1} = \frac{1}{N_i} \sum_{\theta_i} \frac{||V_{es}(\theta_i)| - |V_{ef}(\theta_i)||}{|V_{es}(\theta_i)|}$$

$$C_{M1} = \frac{1}{N_i} \sum_{\theta_i} \frac{||V_{es}(\theta_i)|^2 - |V_{ef}(\theta_i)|^2|}{|V_{es}(\theta_i)|^2}$$

where V_{es} and V_{ef} are the reflection coefficients of an elastic bottom and the equivalent fluid respectively.

The key difference between the new EEFs and the original EEF is that the new versions do not take the phase of their complex reflection coefficients into consideration: they are magnitude based unlike the phase-sensitive original EEF. The comparisons were made via bottom loss vs. grazing angle plots and transmission loss vs. range plots (as seen in Figures 3 and 4). Additionally, a comparison of Oases (a benchmark simulation similar to RAMS) and the EEF methods in simulating single-frequency signals was made for another experimental bottom material. Multi-frequency signals were also simulated as a side comparison of the 3 EEF versions, the Zhang and Tindle equivalent fluid, and a simulation that completely neglects loss of energy to shear waves.

Chapter 4: Results

First, the bottom loss of all three EEF approximation versions were compared to an elastic/benchmark bottom that is characterized as "hard," meaning the bottom has a high density and high shear speed (Figure 3). The angle window 17° - 23° was chosen to generate these EEFs. This means each EEF approximation is meant to accurately mimic the elastic bottom from 17° to 23° . Here the new EEF approximations Met 1 and 2 are indistinguishable from another. Since they more closely mimicked the elastic bottom loss, it appears phase sensitivity is unnecessary in order to do so. It is also obvious that the Zhang and Tindle equivalent fluid approximation is fairly inaccurate; this is expected since their method is not meant to approximate "hard" bottoms.

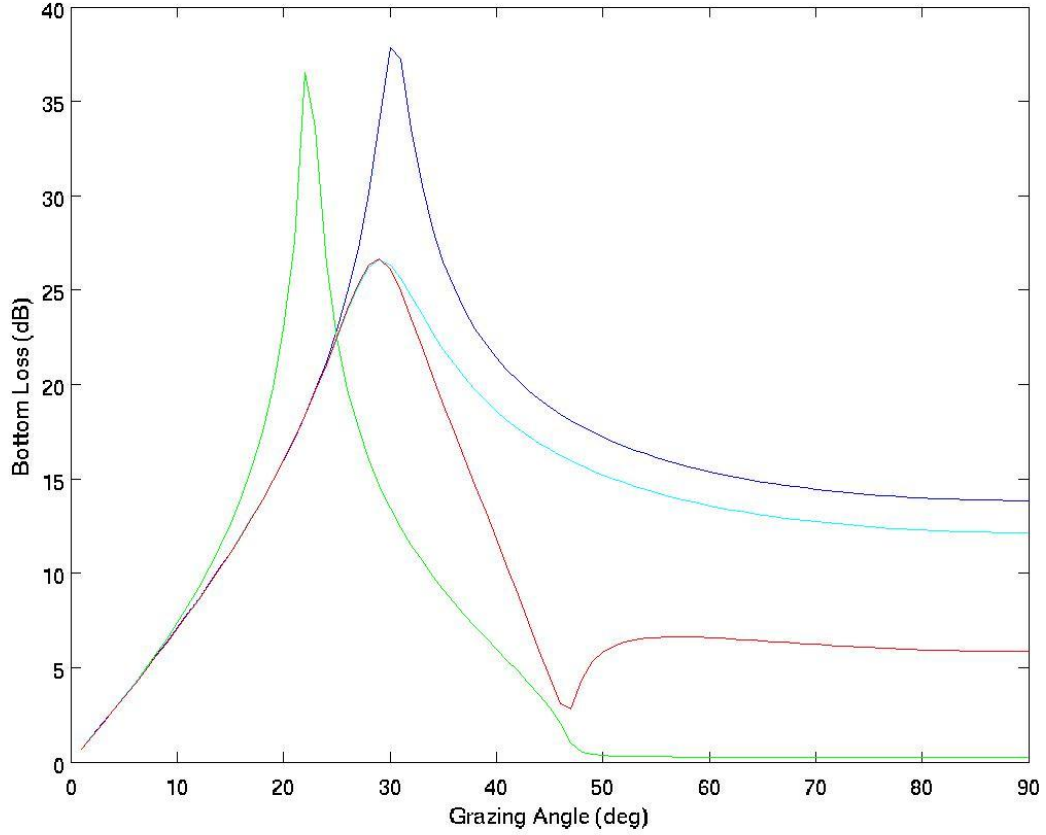


Figure 3—A bottom loss vs. grazing angle plot of a hard elastic bottom (red) compared to the original EEF (blue), Met1 and Met2 (cyan) and the Zhang and Tindle equivalent fluid (green). The hard bottom is characterized by sound speed $c_2 = 2200$ m/s, shear speed $c_s = 1100$ m/s, and density $\rho = 2100$ kg/m³.

Figure 4 depicts another bottom loss vs. grazing angle graph in which the same equivalent fluids are compared to a soft elastic bottom. Again, the EEFs were given a target angle window of 17° to 23°. Here, it is apparent the EEFs more accurately fit the elastic bottom's curve, and again the magnitude based EEFs coincide. Interestingly, the original EEF was able to mimic the jump in bottom loss at around 27° even though it was only supposed to mimic from 17° to 23°; Met1 and 2 neglect the jump entirely but more accurately mimic the elastic bottom loss in the intended interval.

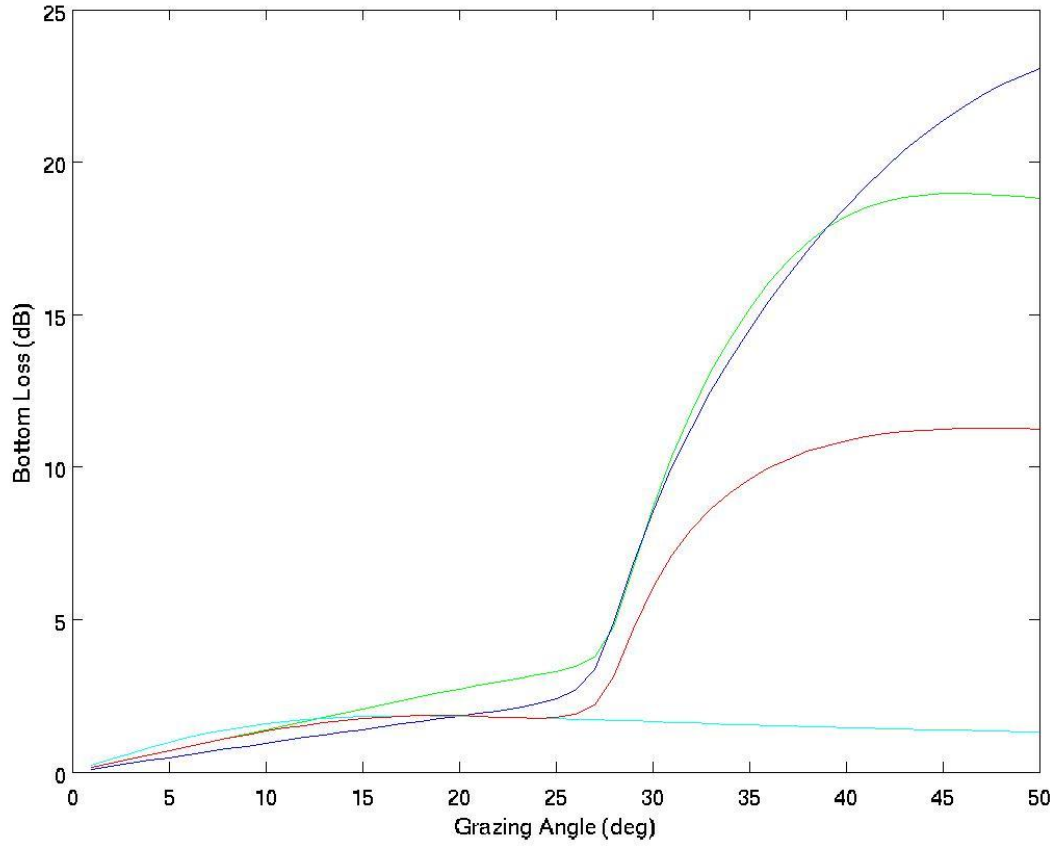


Figure 4—A bottom loss vs. grazing angle plot of a soft elastic bottom (red) compared to the original EEF (blue), Met1 and Met2 (cyan) and the Zhang and Tindle equivalent fluid (green). The soft bottom is characterized by sound speed $c_2 = 1700$ m/s, shear speed $c_s = 600$ m/s, and density $\rho = 1700$ kg/m³.

The following table (Figure 5) depicts the average difference between each equivalent fluid and the elastic bottom's bottom loss curves over the 1° to 45° angle range, including equivalent fluids made to target angle windows 15° to 25° and 5° to 35°.

$\langle \delta BL \rangle$	Original EEF	Met 1	Met 2
"Hard" Bottom			
17° -- 23°	3.94 dB	1.88 dB	1.88 dB
15° -- 25°	3.84 dB	2.89 dB	2.89 dB
5° -- 35°	1.99 dB	1.66 dB	1.66 dB

$\langle \delta BL \rangle$	Original EEF	Met1	Met2
"Soft" Bottom			
17° -- 23°	2.52 dB	3.12 dB	3.12 dB
15° -- 25°	2.28 dB	3.07 dB	3.08 dB
5° -- 35°	1.15 dB	1.00 dB	0.95 dB

Figure 5—A table of average difference values between the bottom loss of EEFs to their respective elastic bottoms over the angle interval of 1° to 45°. The "hard" and "soft" bottoms have the same characteristics as those used in Figure 3 and Figure 4 respectively.

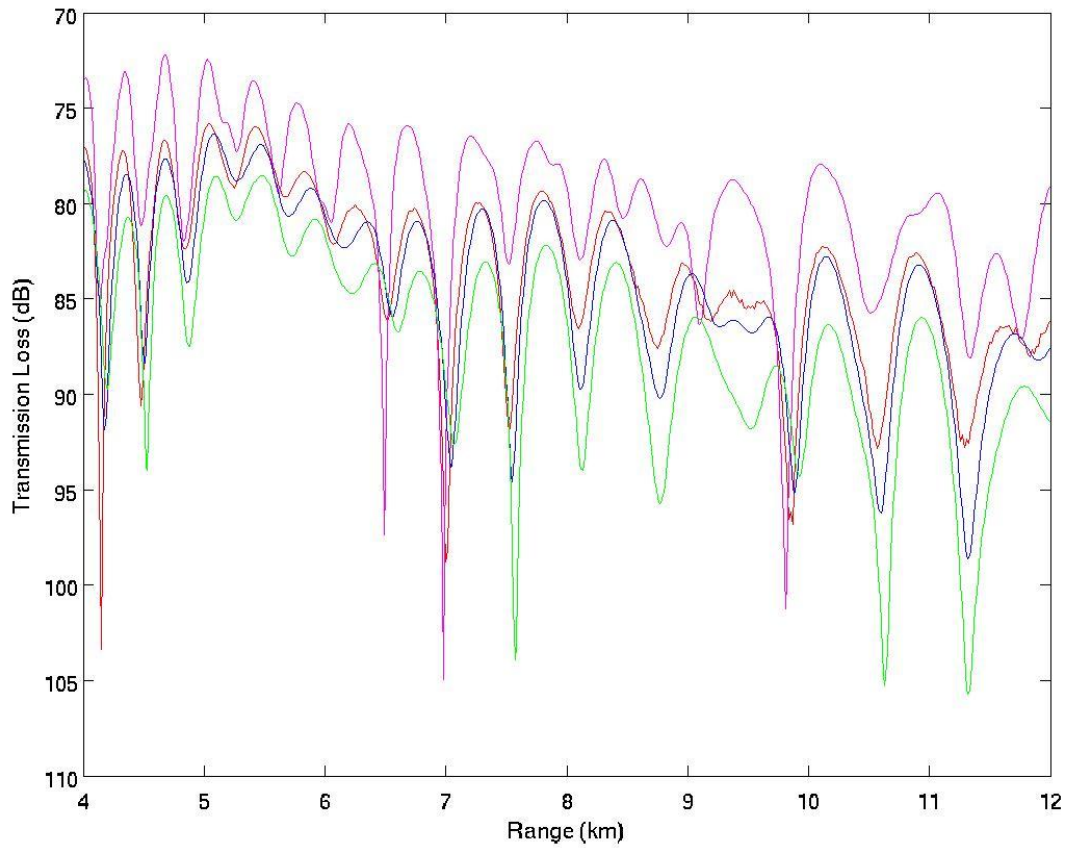


Figure 6—A transmission loss vs. range plot of the Oases benchmark simulation (red) compared to a simulation that neglects shear (magenta) the original EEF (blue), and the Zhang and Tindle equivalent fluid (green). The frequency chosen for this simulation was 75 Hz and the plot depicts transmission loss from 4 to 12 kilometers.

The next basis for comparison is transmission loss vs. range. Generating transmission loss plots can be computationally involved, so we will choose to run single-frequency simulations--75Hz was chosen. In Figure 6, transmission losses are plotted for Zhang and Tindle's equivalent fluid, the original metric with target angle window 10° to 30° , a run that neglects loss of energy to shear, and the benchmark simulation. The transmission loss axis is inverted for comprehension's sake: the higher a signal is on the plot, the less is lost in transmission, and therefore the louder the signal is. Here it is clear and sensible that the run that neglects shear loses the least amount of transmission and is therefore higher on this inverted transmission loss axis. It is also apparent the phase-sensitive EEF better mimics the benchmark than Zhang and Tindle's approximation.

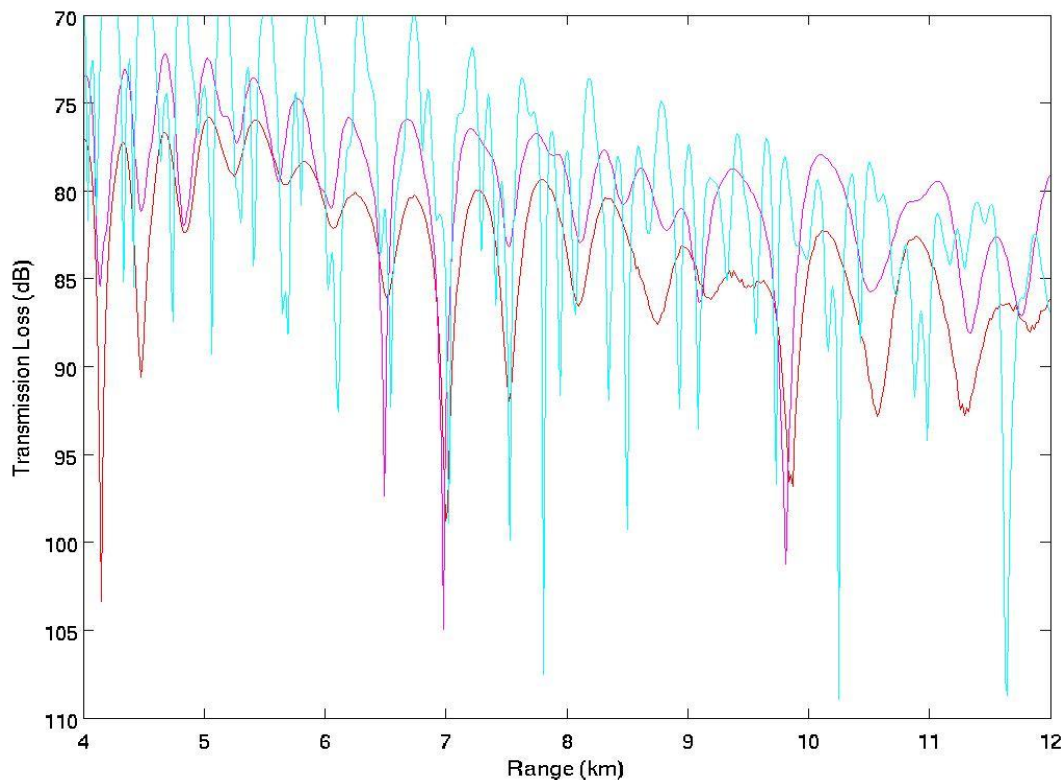


Figure 7—A transmission loss vs. range plot of the Oases benchmark simulation for the 1700/600/1700 soft bottom (red) compared to a simulation that neglects shear (magenta), and the EEFs Met 1 and Met 2 with target angle window 10° to 30° (cyan). The frequency chosen for this simulation was 75 Hz and the plot depicts transmission loss from 4 to 12 kilometers.

However, the same cannot be said for the magnitude-based EEF approximations, as seen when their transmission losses are added to the plot (Figure 7). The original EEF and Zhang and Tindle's approximation are removed for clarity. Both EEFs' transmission losses are nearly identical as earlier in bottom loss, but they clearly are incapable of accurately simulating single frequency signals.

Additionally, multi-frequency simulations were run to compare the different approximations. These are soft-bottom acoustic simulations done via depth vs. time plots and include intensity measurements through color from blue to red. Figure 8 is an example plot of a simulation that neglects shear.

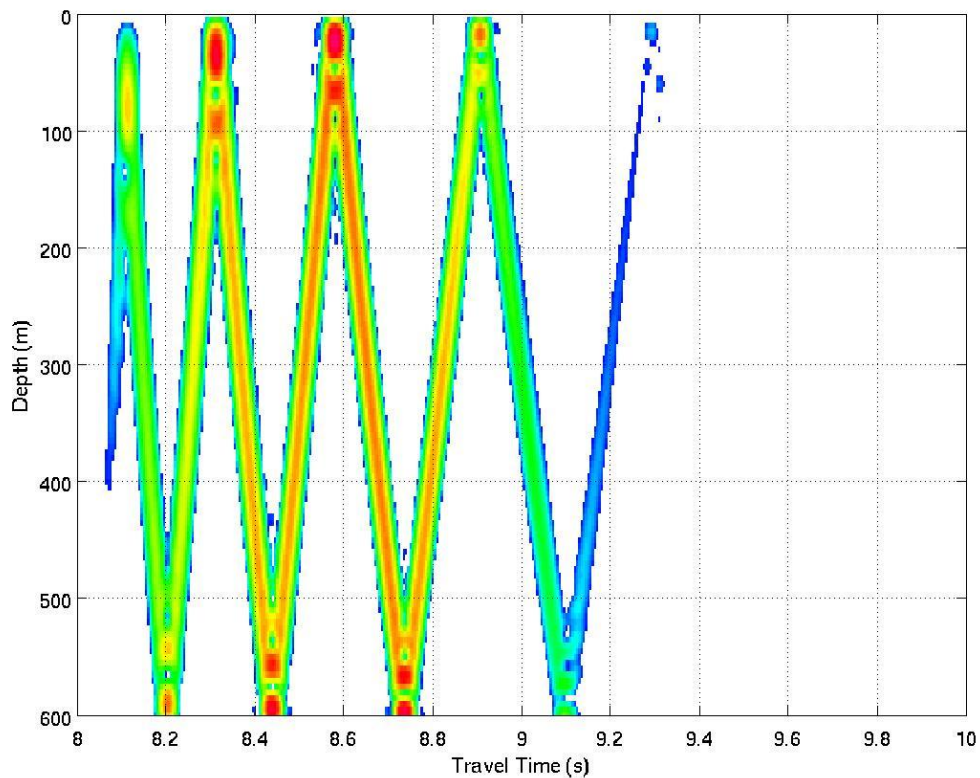


Figure 8—A depth vs. time plot of a multi-frequency simulation that neglects shear. This simulation is done for a 1700/600/1700 soft bottom, and the band of frequencies simulated are from 25 Hz to 75 Hz.

Figure 9 depicts Zhang and Tindle's approximation in a multi-frequency simulation using the same intensity scale of Figure 8. (All further depth vs. time plots will use this scale). Figure 10 depicts the multi-frequency simulations of the original EEF approximation. Similar to the single-frequency simulations, the original EEF lost less energy in bottom interaction than Zhang and Tindle's approximation. Figure 11 shows the multi-frequency simulations for the Met 1 approximation. It appears to be about as accurate as the simulation neglecting shear, but this is only a preliminary simulation: the method to simulate multi-frequency bottom interaction with a magnitude based EEF is still in its experimental stages.

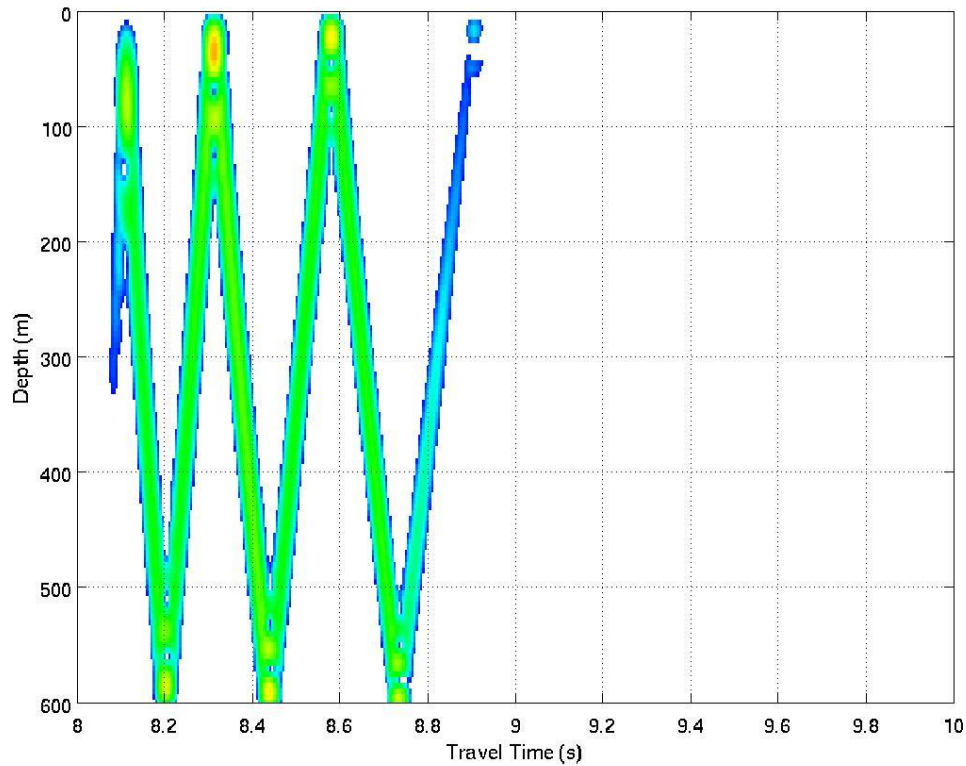


Figure 9—A depth vs. time plot of a multi-frequency simulation using Zhang and Tindle's equivalent fluid approximation. This simulation is done for a 1700/600/1700 soft bottom, and the band of frequencies simulated are from 25 Hz to 75 Hz.

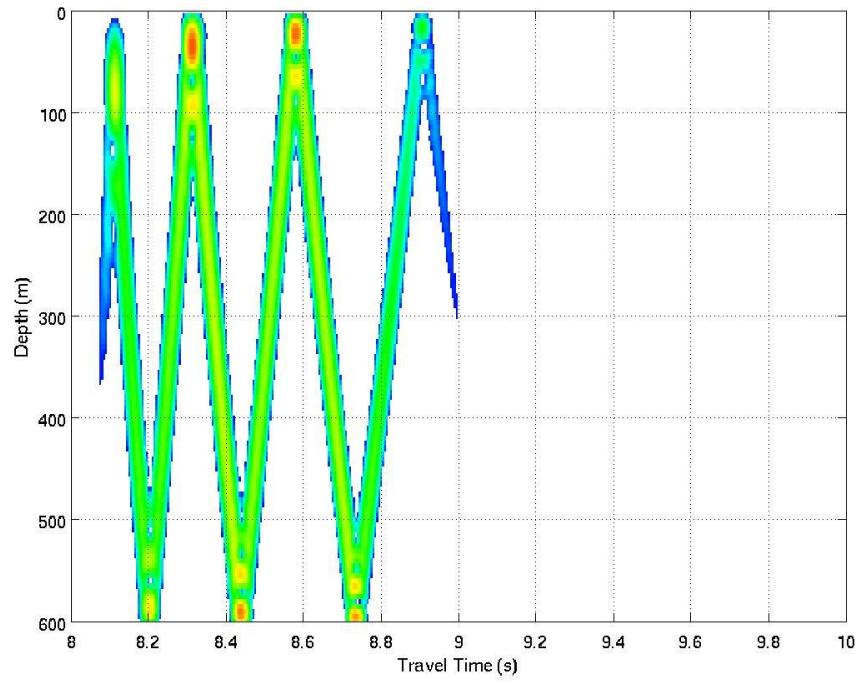


Figure 10—A depth vs. time plot of a multi-frequency simulation using the original EEF approximation. This simulation is done for a 1700/600/1700 soft bottom, and the band of frequencies simulated are from 25 Hz to 75 Hz.

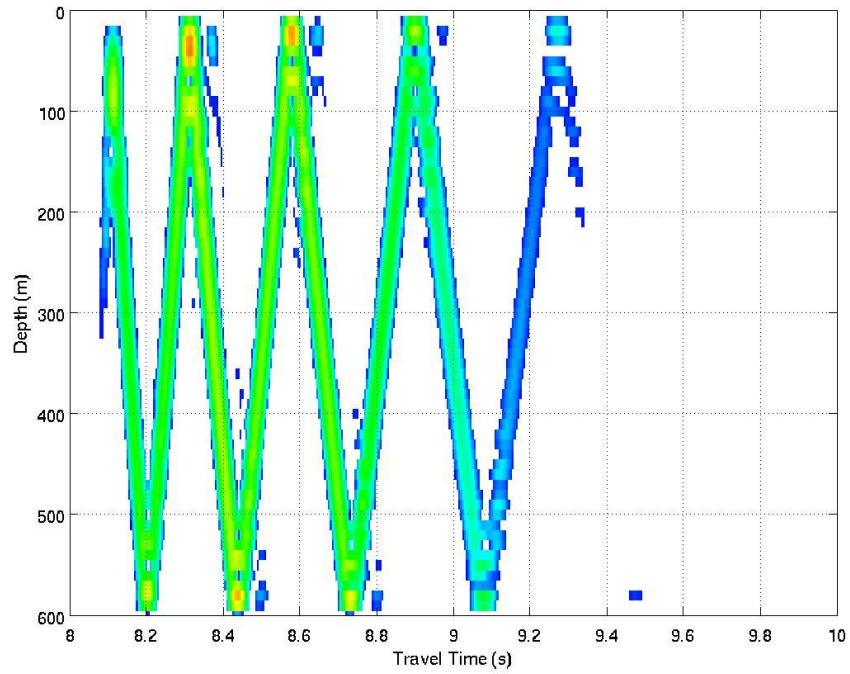


Figure 11—A depth vs. time plot of a multi-frequency simulation using Met 1. This simulation is done for a 1700/600/1700 soft bottom, and the band of frequencies simulated are from 25 Hz to 75 Hz.

Chapter 5: Discussion

It is clear there are various advantages and disadvantages in a metric choice for approximating Expanded Equivalent Fluids. In the way of mimicking a realistic bottom's reflection coefficient (as seen in the bottom loss plots), phase-sensitivity is not needed and lessens the accuracy of the EEF approximation. However, it appears to be paramount in order to run actual acoustic simulations. It is speculated that Met 1 and Met 2 are unable to simulate single frequency transmission since their lack of phase sensitivity causes transmission interference. Testing the new metrics in multi-frequency simulations has yet to yield promising results, but research in this area is still underway.

As an aside, it is worth noting that the original EEF outperformed Zhang and Tindle's equivalent fluid on every front; since the former is a more parameterized version of the latter, this makes sense. In addition, the original EEF approximation has shown a capacity to handle fast shear speeds and intervals of large grazing angle unlike Zhang and Tindle's equivalent fluid approximation. This metaphorically "opens the door" into new areas of use for equivalent fluids in which Zhang and Tindle's method was found unviable.

Dr. Vera's Expanded Equivalent Fluid approximation has proven to be quite versatile. For a particular range of grazing angles, it can accurately mimic a bottom and run a variety of acoustic simulations, which is very useful in the interpretation of tomographic data. Both the types of bottoms and range of angles have shown to reach large limits (5° to 35° for the angle range) without yielding inaccurate results, and it remains unmatched by other methods of approximating bottom interaction.

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